FORMAL ASPECTS OF QUANTUM INFORMATION IN HIGH ENERGY PHYSICS

Based on white paper: "Quantum information in quantum field theory and quantum gravity", with Tom Hartman, Matt Headrick, Mukund Rangamani, and Brian Swingle

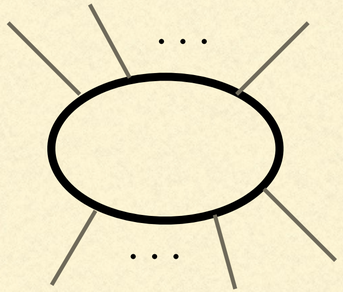
Thomas Faulkner, UIUC

TRADITIONAL QFT

- Traditional QFT interested in correlation functions:

$$\langle \Omega | \, \phi_1 \phi_2 \ldots \phi_k \, | \Omega \rangle$$

- From which we derive, S-matrix, response functions, etc.



$$G_R(t) = \theta(t) \, \langle [\phi(t), \phi(0)] \rangle$$

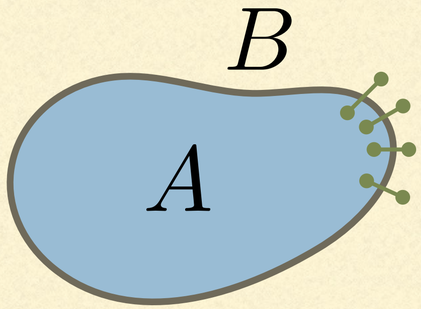
- Inherently quantum aspects are often hidden: such as Entanglement, scrambling …

ENTANGLEMENT IN QFT

- Two parties entangled if wave function:

$$\psi_{AB} \neq \psi_A \otimes \psi_B \qquad\qquad S_{EE}(A) = S_{vN}(\rho_A)$$

- In QFT locality of interactions implies the most interesting split is spatial:



$$S_{EE(A)} \sim \frac{\text{Area}(\partial A)}{\epsilon^{d-2}}$$

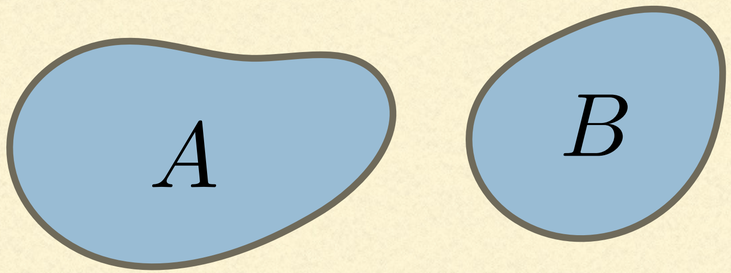
[Sorkin, Bombelli et al. , Srednicki, Holzhey et al.]

- Divergent entanglement! Due to short distance correlations, adiabatic vacuum

UV FINITE QUANTITIES

- Two UV finite quantities have played significant role in the theory:

Two regions:



$$I(A:B) = S_{EE}(A) + S_{EE}(B) - S_{EE}(AB)$$

Strong-Subadditivity:

$$I(AC:B) \geq I(A:B)$$

Two states:

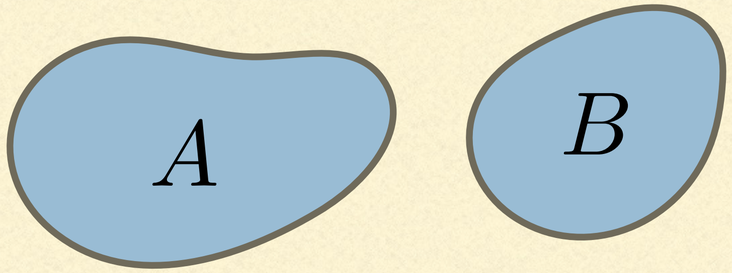
$$S_{\rm rel}(\rho_A|\sigma_A) = {\rm Tr}\rho_A(\ln\rho_A - \ln\sigma_A)$$

Data processing:

$$S_{\rm rel}(\rho_{AB}|\sigma_{AB}) \geq S_{\rm rel}(\rho_A|\sigma_A)$$

CHANGE OF PERSPECTIVE

- Information perspective, characterizes instead total correlations:



$$2I(A:B) \geq \left(\frac{\langle O_A O_B \rangle_c}{\|O_A\| \|O_B\|} \right)^2$$

[Wolf et al.]

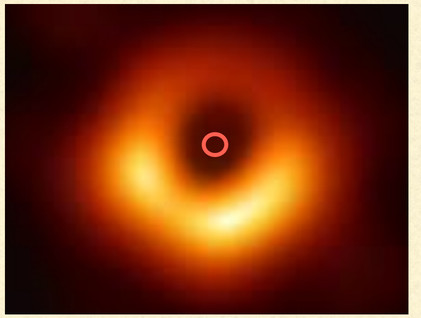
- Eschews specific field content: field redefinition invariant, duality frame invariant,
- Opens up new constraints on QFT - Q.I. bounds, such as SSA, data processing etc.

CHANGE OF PERSPECTIVE - BLACK HOLES

- Forced on us by black hole thermodynamics:

$$S_{BH} = \frac{\text{Area}(H)}{4G_N}$$

[Bekenstein, Hawking]



- What are the microscopic UV degrees of freedom? QI doesn't care, gives IR constraint on UV theory

- Area divergence and black hole area related: $S_{\text{gen}} = S_{BH} + S_{EE}(\text{out})$

[Bekeinstein; Wall]

DEVELOPING TOOLS FOR CHARACTERIZING ENTANGLEMENT IN QFT

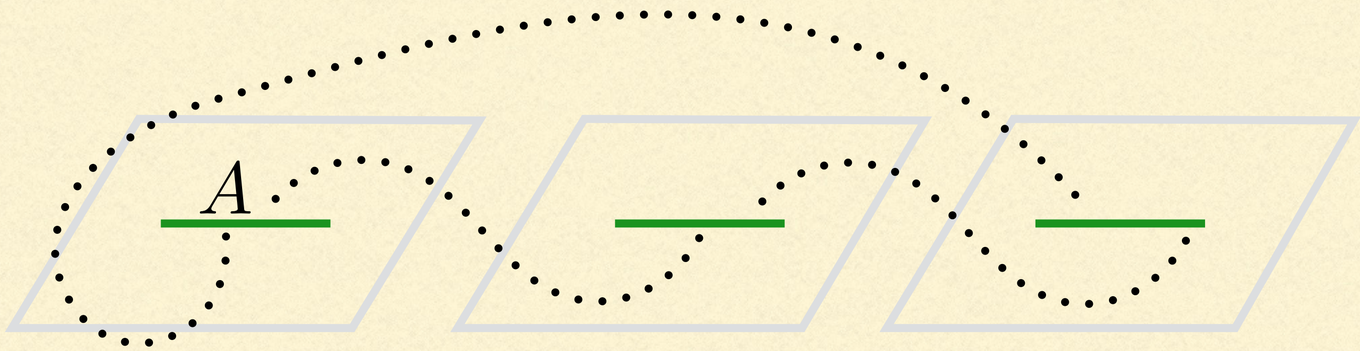
DEVELOPING TOOLS

- Much progress over last 10 years on developing and refining toolkit for QI in QFT
- Broadly speaking:
  - Path integral methods (replica trick)
  - Symmetries (spacetime, CFT)
  - Free theories (correlation matrix technique)
  - AdS/CFT
  - Algebraic QFT
- One goal: relate to more traditional QFT correlation functions (new constraints?)

DEVELOPING TOOLS

- Path integrals and the replica trick:

$$\mathrm{Tr}_A \rho_A^n = Z(\mathcal{M}_n) =$$



[Larsen et al. , Cardy, Calabrese]

$$S_{EE}(A) = \lim_{n\to 1} (n-1)^{-1} \ln \mathrm{Tr}\rho_A^n$$

- CFT twist operators:

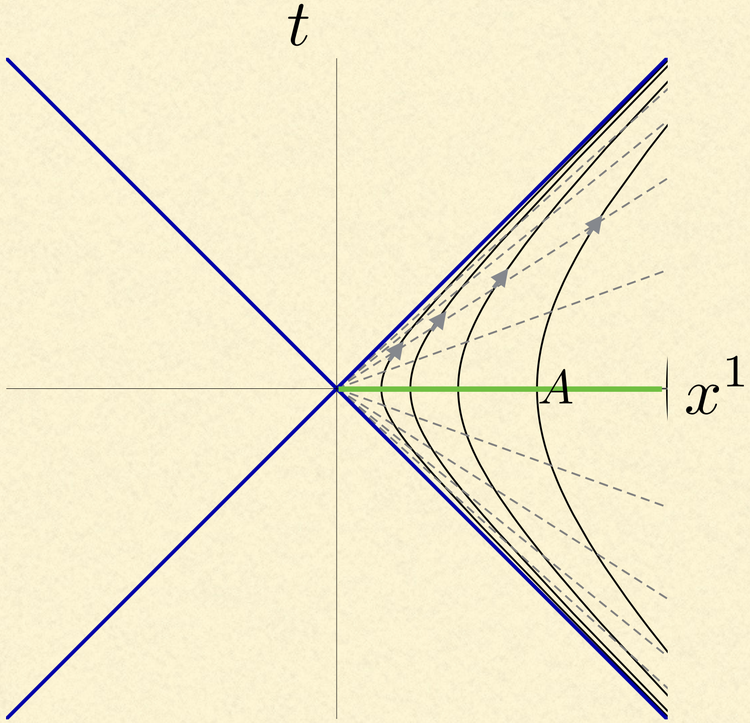
$$Z(\mathcal{M}_n) = \langle \sigma_n^\dagger(a_1)\sigma_n(a_2)\rangle_{CFT^{\otimes n}}$$

Applications: 2d CFTs, gravitational entropy, replica wormholes ...

[Headrick; TF; Lewkowycz, Maldacena ..]

DEVELOPING TOOLS

- Bisagnano-Wichmann, Unruh, Rindler and spacetime symmetries:



In some cases can write down full density matrix:

$$-\ln \rho_A = \int_A x^1 T_{00}$$ = gen. half sided boost

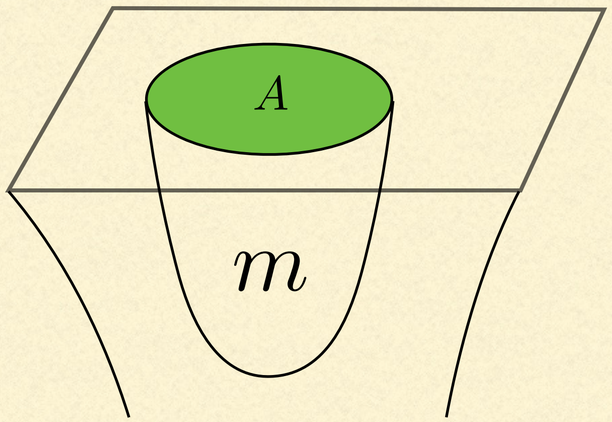
Full boost, less singular = full modular Hamiltonian

Applications: black hole thermo, perturbation theory, ANEC, Einstein's equation from entanglement

[Wall; Rosenhaus et al, TF, Parrikar, Leigh; van Raamsdonk et al. …]

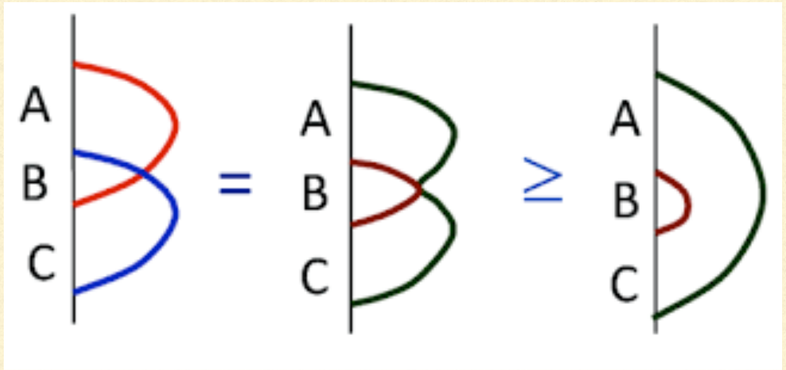
DEVELOPING TOOLS

- AdS/CFT:



$$S_{EE}(A) = \min \mathrm{ext}_{\mathrm{m}\sim\mathrm{A}} \frac{\mathrm{Area}(m)}{4G_N}$$

SSA: [Headrick, Takayanagi; Wall]



[Ryu-Takayanagi; Hubeny, Rangamani, Takayanagi]

Dynamical setting: SSA requires Einstein's equations [van Raamsdonk et al.]
+ NEC. Partial converse: Einstein's equations from entanglement constraints

Applications: too many to mention

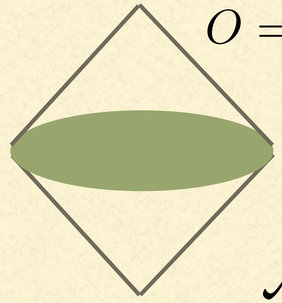
DEVELOPING TOOLS

[Haag, Kastler]

- Algebraic QFT, redux: algebraic QFT is in some sense the correct language to discuss QI in QFT:

$$O \rightarrow \mathcal{A}(O) = \mathcal{A}(A)$$

A_c



$O = \mathcal{D}(A)$

$O' = \mathcal{D}(A_c)$

$\mathcal{A}(O)' = \mathcal{A}(O')$

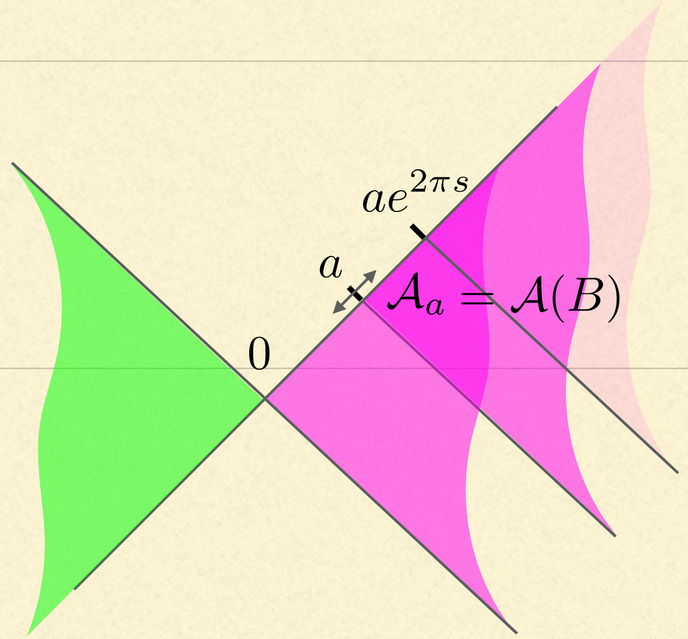
- Von Neumann algebra: $\text{type} - \text{III}_1$ compared to: $\text{type} - \text{I}$ standard type for QM

- Quantum Info: modular theory, defining continuum limit: $\Delta = \rho_A \otimes \rho_{A_c}^{-1}$ [Araki]

Defining entanglement entropy not an option …

[Witten]

DEVELOPING TOOLS

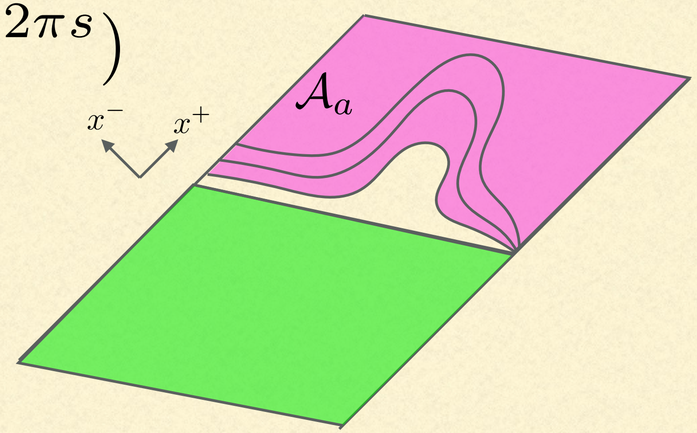


- Half-sided modular inclusion: [Wiesbroch]

$$\Delta_A^{is}\mathcal{A}(B)\Delta_A^{-is} \subset \mathcal{A}(B) \qquad s \geq 0$$

Basic causality constraint gives: $\Delta_A^{is}U_+(a)\Delta_A^{-is} = U_+(ae^{2\pi s})$

$$U_+(a) = \exp(iaP_+) \qquad P_+ \geq 0$$



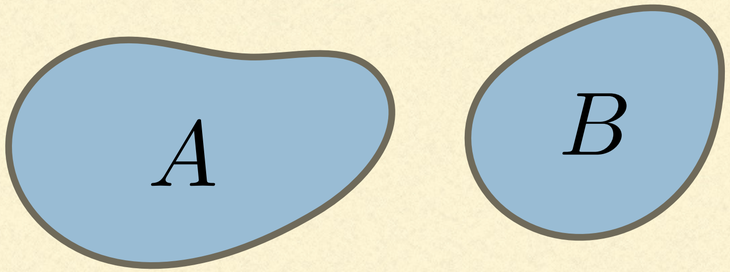
Reconstruct relativistic symmetries from QI

Applications: Proofs of energy conditions. Emergence of spacetime from entanglement

[Casini et al; Ceyhan, TF] [Min, TF, Wang; Leutheusser, Liu]

DEVELOPING TOOLS

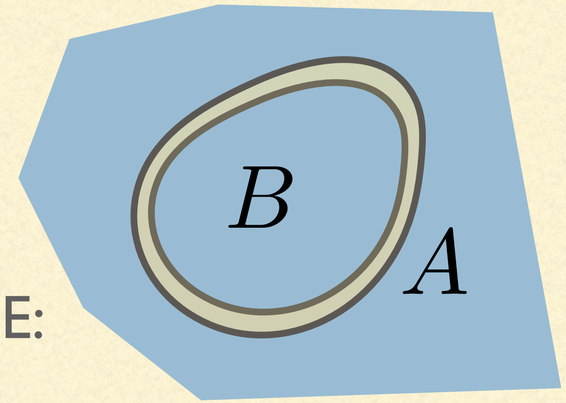
- Split property:



$$\mathcal{A}(B) \subset \mathcal{N} \subset \mathcal{A}(A)'$$
$$\text{type} - \text{I}$$

[Haag; Doplicher-Longo]

- Used to define: mutual information (split state)
- Entropy of type-I factor well defined - Reflected entropy, proxy for EE:



[Longo et al., Casini et al., Hollands et al. ….]

Applications: rigorous computations of relative entropy, mutual information.
Reflected entropy: a new QI measure (tripartite), EW cross section in AdS/CFT

[Dutta, TF]

DEVELOPING TOOLS - OPEN QUESTIONS

- Didn't mention: Mixed state entanglement, negativity

  [Calabrese, Cardy, Tonni; Hollands, Sanders; …]

- Entanglement in string theory

  [Susskind, Uglum; Dabholker; He et al; Balasubramanian et al.; Hubeny et al; Witten]

- Beyond von Neumann entropy - one shot entropies, non-asymptotic quantities, entanglement measures

  [… Akers, Penington]

- What is the replica trick computing in gravitational theories? [regularized entropy?]

  [Renner, Wang]

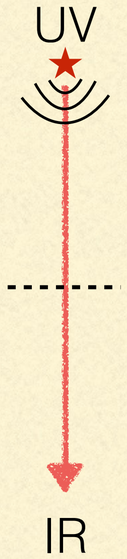
- Approximate half-sided modular inclusions? Beyond the chaos bound?

  [Chandrasekaran, TF, Levine]

RENORMALIZATION **G**ROUP **FLOWS**

RG FLOWS

- Flows between fixed points in QFT, ``degrees of freedom" lost in the IR



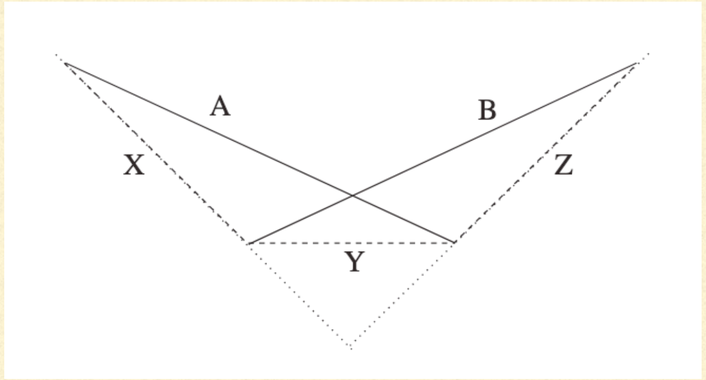
- c-theorem in 2d [Zamolodchikov]
- F-theorem in 3d
- a-theorem in 4d [Komargodski, Schwimmer]

Proofs in 2d,4d using ``traditional QFT" correlation functions. Also AdS/CFT proofs

- Can't reverse the flow. Why? C-theorems: now proven using entanglement entropy

RG FLOWS

- Input: relativistically invariant vacuum, SSA. In 2d: [Casini, Heurta]



$$S_{EE}(XY) + S_{EE}(YZ) \geq S_{EE}(Y) + S_{EE}(XYZ)$$

$$C(r) = rS'(r) \qquad C'(r) \leq 0$$

r

IR $\quad r \to \infty$

UV $\quad r \to 0$

- Higher dimensions (3d,4d) trickier

[Casini, Heurta; Casini, Test, Torroba]

- Intriguing underpinning: vacuum of CFT is a Markov state on a lightcone, connections to QEC codes? [Lashkari]

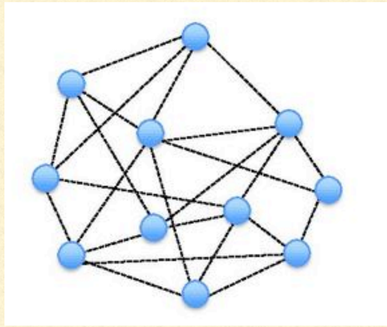
DYNAMICS

SCRAMBLING AND CHAOS

- Based on Hayden-Preskill thought experiment to extract information efficiently from an old Black hole, Sekino-Susskind conjectured a bound on time scale for scrambling of QI:

$$t > t_s = \frac{\beta}{2\pi} \ln S$$

Scrambling time: perturbation to system delocalized over entire system



Saturated in SYK model at large J and holographic theories

[Maldacena, Shenker, Stanford]

- Growth of thermal commutators: determined by OTOC:

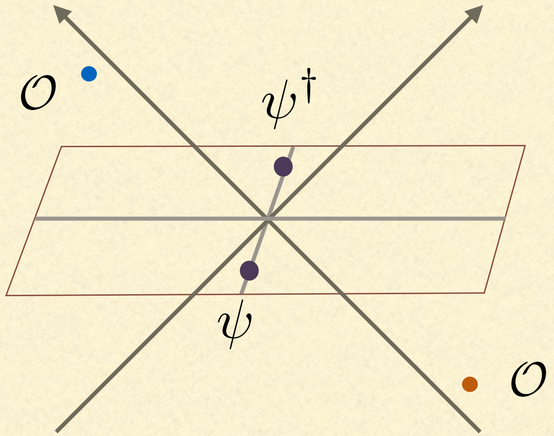
$$-\left\langle [\psi(0), \mathcal{O}(t)]^2 \right\rangle_\beta \qquad F(t) = 1 - \#N^{-1} e^{\lambda_L t}$$

Analyticity + Cauchy-Schwarz

$$\lambda_L \leq \frac{2\pi}{\beta} \qquad \# > 0$$

OTOC'S AND THE ANEC

- If we take the time evolution to be determined by Rindler time (boosts) then same argument constraints vacuum correlation functions



$$f(u) \propto \langle\psi|\, \mathcal{O}(u,v)\mathcal{O}(-u,-v)\, |\psi\rangle$$

$$|\psi\rangle = \psi\,|\Omega\rangle$$

[Hartman, Kundu, Tajdini]

Kinematic limit: lightcone OPE:

$$f = 1 - \#e^{2\pi t}\,\langle\psi|\int T_{++}dx^+\,|\psi\rangle$$

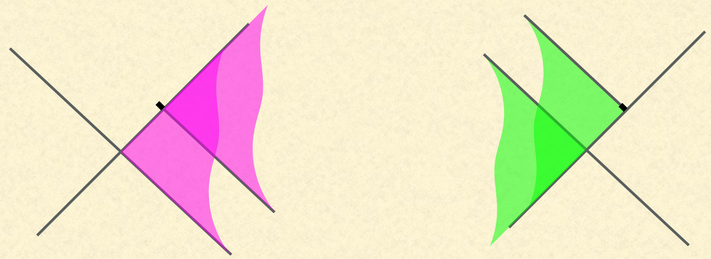
- NEC, $\langle T_{++}\rangle \geq 0$ violated quantum mechanically. Averaged over null line in Minkowski space

ANEC AND RELATIVE ENTROPY

- There is another way to prove the ANEC, using directly quantum information:

[Faulkner, Leigh, Parrikar, Wang]

$$\int dx^+ \langle\psi| T_{++} |\psi\rangle = \Delta S_{\rm rel}(\psi|\Omega) - \Delta S'_{\rm rel}(\psi|\Omega)$$



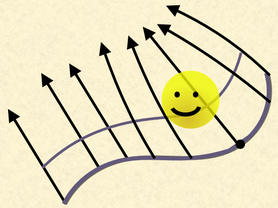
- ANEC follows then from data processing

- Now understand as a consequence of half-sided modular inclusion algebra

[Casini, Test, Torroba]

QNEC AND BEYOND

- In gravity focusing of light by matter: $\langle T_{++} \rangle \geq 0$. But violated quantum mechanically



- Quantum focusing considers focusing of a null hyper surface but replaces the area by

$$A \to A + 4G_N S_{EE}(\text{out})$$

[Bousso, Fisher, Leichenauer, Wall]

- Conjectured property of quantum gravity, implies a new bound on QFT without gravity:

$$\text{QNEC } \langle T_{++} \rangle \geq \frac{1}{2\pi\delta a} \frac{d^2 S_{EE}}{(dx^+)^2}$$

General proof: relative entropy and half sided modular inclusions

[Balakrishnan et al; Ceyhan, TF]

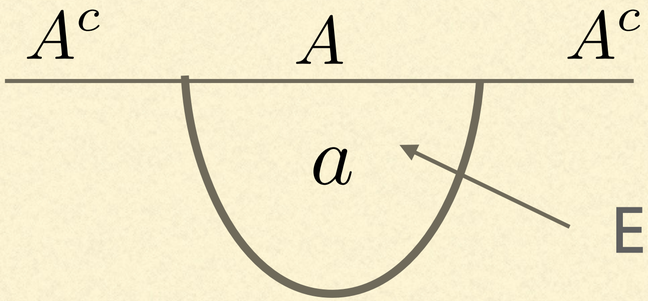
QI IN GRAVITY

QI IN GRAVITY

- Ryu-Takayanagi - biggest revolution in our understanding of QGravity in past 20 years
- Really a statement about quantum error correction and what regions of the bulk are reconstructable from the boundary:

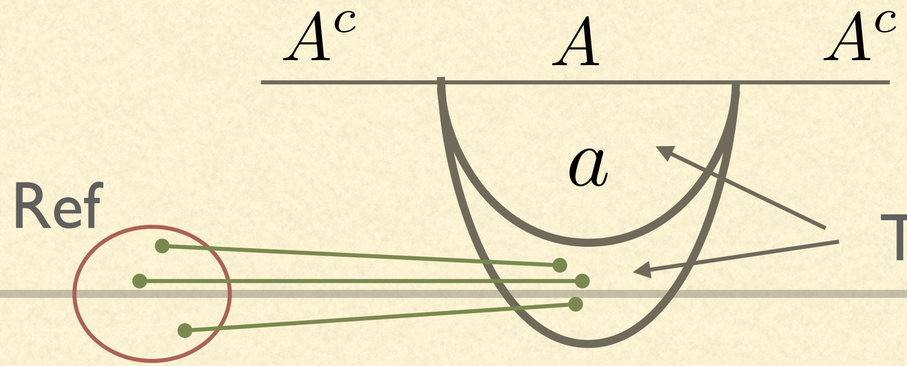
$$S_{EE}(A) = \frac{\text{Area}(\partial a)}{4G_N} + S_{EE}(a) + \dots$$

[Almheiri et al., Dong, et al., Harlow]



Entanglement wedge

- New developments in QEC codes due to gravity (state-dependent entanglement wedges)



Two candidates

Alpha-bits, state-specific unitary reconstruction, …

[Hayden, Penington; Akers, Penington]

QI IN GRAVITY

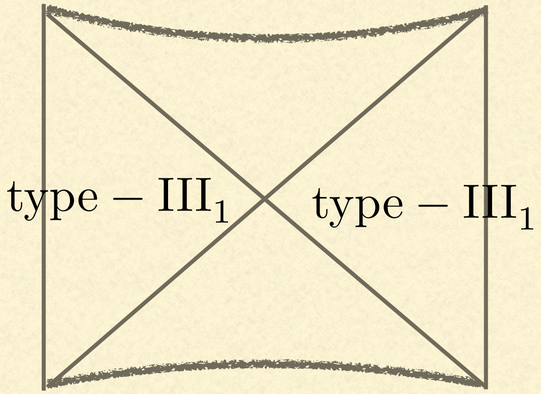
- New results on non-trivial von Neumann algebras emerging in the bulk of AdS/CFT from the large-N limit of boundary theory

[Leutheusser, Liu]

N=4 SYM above Hawking-Page transition, $N \to \infty$

$$|TFD\rangle = \sum_E e^{-\beta E/2} |E\rangle_{CFT_L} \otimes |E^\star\rangle_{CFT_R}$$

$$S_{EE} = \frac{A}{4G_N} \to \infty$$



Quantum corrected, energy fluctuations:

$\text{type} - \text{II}_\infty$

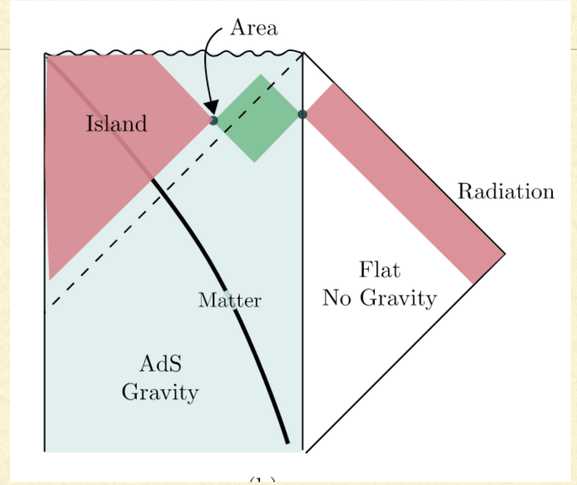
[Witten]

- Certainly should be related to QEC paradigm

QI IN GRAVITY

- Island rule, moves lessons of Ryu-Takayanagi beyond AdS/CFT:

$$S_{EE}(\rho_R) = \min \operatorname{ext}_I \left(\frac{\text{Area}(\partial I)}{4G_N} + S_{EE}(\tilde{\rho}_{R\cup I}) \right)$$



[Pengington; Almheiri et al.]

- Similarly, non-trivial von Neumann algebras appear in setups beyond AdS/CFT, notably de Sitter space static patch (type-II_1) and flat space black holes

[Chandrasekaran et al.]

- Ideas developed from AdS/CFT used for more general purposes. Opens up possibilities to study cosmology, entropy of de Sitter Horizons etc beyond AdS/CFT. Next SNOWMASS!

THANK YOU